

NetzCope: A Tool for Displaying and Analyzing Complex Networks

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Networks are a natural and popular mechanism for the representation and investigation of a broad class of systems. But extracting information from a network can present significant challenges. We present NetzCope, a software application for the display and analysis of networks. Its key features include the visualization of networks in two or three dimensions, the organization of vertices to reveal structural similarity, and the detection and visualization of network communities by modularity maximization.

1. Introduction

Networks describe the structure and dynamics of relations between objects or agents: in short, networks are everywhere. In biology they have become a prominent subject of research, in engineering network concepts have become ever more important—from Kirchhoff’s laws in the 19th century to optimizing the design of a microchip to-day—and in the socio-economic field “globalization” is paradigmatic for their dominance. A short list of examples may serve to underscore this point:

Neural Networks The well-studied nematode *Caenorhabditis elegans* has a neural network of some 300 neurons with some 7000 connections between them.

Metabolic Networks metabolic processes in the cell.

Protein interaction networks physical interactions between an organism’s proteins.

Transcriptional networks regulatory interactions between different genes.

Food webs Who eats whom?

Sexual relations and infections AIDS epidemiological models

Pollination networks plants and their pollinators

Electric networks stability of power grids

Electronic networks, computer chips computing speed

Airline networks service efficiency

Internet search engines

Linguistics words linked by co-occurrence, language families

Social networks identification of central players, gate keepers,

Collaborations actors, authors, research labs, ...

Typically these networks exhibits considerable complexity and more often than not their structure is far from transparent.

In this work, we present *NetzCope*, a software application for the display and analysis of complex networks. NetzCope is a general purpose tool for investigating networks, allowing the user to interactively explore networks, especially with regard to visualizing the most important relationships in the network.

1.1. *Bipartite Networks*

The NetzCope software was originally developed to find and display the structure hidden in long lists of tens of thousands of collaborative research projects sponsored by the European Union. Said networks are *bipartite*, with links always connecting members of two different sets. Some examples:

Regulatory networks transcription factors and target genes

Economic networks Financial centers and multinationals, firms and board members

Collaboration networks actors and films in which they appeared together, laboratories and joint research projects, scientists and joint publications, ...

1.2. *Modularity*

Of particular interest to the exploration of the network of EU-funded projects—and, indeed, to networks in general—is any possible modular structure of the network. Quoting from an article on network biology:

Cellular functions are likely to be carried out in a highly modular manner. In general, modularity refers to a group of physically or functionally linked molecules (nodes) that work together to achieve a (relatively) distinct function. Modules are seen in many systems, for example, circles of friends in social networks or websites that are devoted to similar topics on the World Wide Web. Similarly, in many complex engineered systems, from a modern aircraft to a computer chip, a highly modular structure is a fundamental design attribute. Biology is full of examples of modularity... (Barabási and Oltvai¹)

To reach a more precise understanding, in order are a few words on basic concepts of graph theory, the mathematical formulation of networks.

2. A Few Words on Graphs

Let

- V be a set (*vertices*)
- E be a set of vertex pairs from $V \times V$ (*edges*).

The pair $G = (V, E)$ is called a graph. Given a partition $V = V_1 + V_2$. If there are no edges between pairs of points within either V_i , then G is called *bipartite*.

The number of edges of a vertex v we call the *degree*:

$$d_v = \# \{(v, \cdot) \in E\}$$

A simple graph $G = (V, E)$ is described by an adjacency matrix indicating whether vertices i and k are connected by an edge:

$$a_{ik} = \begin{cases} 1 & i \sim k \\ 0 & \text{otherwise} \end{cases}$$

The degree of vertex k is then

$$d_k = \sum_i a_{ik}$$

and we shall set

$$D = \text{diag} \{d_1, \dots, d_n\}.$$

The Laplacian

$$L = D - A$$

and the normalized Laplacian

$$\mathcal{L} = 1 - D^{-1/2} A D^{-1/2}$$

play a central role. In particular, $-\mathcal{L}$ is (up to a similarity transformation) the generator of a continuous time random walk on the graph, with equal probability $1/d_k$ along each edge.

2.1. How to plot a graph

Typically a graph or network will be given simply as a list of agents (i.e., vertices) and their relations (i.e., edges). How would one translate such a list into a graphical display?

To begin with, try to arrange the vertices on a straight line: put each vertex k at position x_k such that those connected by an edge will be as close as possible, as if connected by elastic springs. Mechanics tells us that such an arrangement would minimize the expression

$$E = \frac{1}{2} \sum a_{ik} (y_i - y_k)^2.$$

Neurons in the nematode *C. elegans* are said to be distributed in this fashion!

We can also write this as the scalar product

$$E = (x, (D - A)x).$$

Of course the minimum $E = 0$ would arise for the vector $x = e_0$ with $x_1 = x_2 = \dots x_k = \dots = 1$, indeed

$$(D - A)e_0 = (D - A) \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = 0.$$

Here all the vertices are at the same place, $x_k = 1$. Excluding this, we are led to the next eigenvector of $L = D - A$ with

$$(D - A)e_1 = \lambda_1 e_1$$

In practice a better ordering is achieved using $D^{1/2} f_1$, where f_1 is the Fiedler vector, the eigenvector of the normalized Laplacian corresponding to the smallest positive eigenvalue.

2.2. Modularity of Graphs

A good division of a network into communities is not merely one in which there are few edges between communities; it is one in which there are fewer than expected edges between communities. (M. E. J. Newman²).

A popular measure of the quality of such a division or decomposition is the *modularity*.³ Modularity is—up to a normalization constant—the number of edges within communities c minus those for a null model:

$$Q \equiv \frac{1}{2|E|} \sum_c \sum_{i,j \in c} (A_{ij} - P_{ij}),$$

where $|E|$ is the number of edges or links, and

$$P_{ij} \equiv \frac{d_i d_j}{2|E|}$$

corresponds to a random graph model with a fixed set of vertices and the constraint that on average they should reproduce a given distribution of vertex degrees d_i .⁴

In empirical investigations, modularity values above roughly 0.3 are indicative of a partitioning of the network vertices showing significant modular structure. Modularity close to one would correspond to a near perfect decomposition of the network into loosely interconnected communities.

The goal now is to find a division of the vertices into communities such that the modularity Q is maximal. An exhaustive search for such a decomposition is out of the question: even for moderately large graphs there are far too many ways to decompose them into communities. But fast approximate algorithms do exist,⁵ many based on the idea of greedily merging small communities into larger ones with a higher modularity.^{6,7}

For bipartite graphs the null model must be modified, to reproduce the characteristic form of bipartite adjacency matrices

$$A = \begin{pmatrix} 0 & B \\ B^T & 0 \end{pmatrix}$$

also for the null model.⁸ This gives a bipartite modularity Q_B . Comparatively few algorithms have been proposed for maximizing Q_B , but methods for unipartite networks can often be adapted with little trouble.

Recently, Barber⁸ proposed an appropriate algorithm (BRIM: bipartite, recursively induced modules) to find communities for bipartite networks. Starting from a (more or less) ad hoc partition of the vertices of type 1, it

is straightforward to optimize a corresponding decomposition of the vertices of type 2. From there, optimize the decomposition of vertices of type 1, and iterate. In this fashion, modularity will increase until a (local) maximum is reached. NetzCope allows to combine a suitable greedy algorithm with BRIM, significantly enhancing the performance of the former.

Modularity has some limitations that should be kept in mind. The measure has a resolution limit dependent on the number of edges in the network, so that small communities cannot be found in large networks by simply finding a maximum in the modularity.⁹ Further, modularity maximization is an NP-complete problem;¹⁰ typical to the class, there are exponentially many local maxima in Q , so some ambiguity of decompositions is inevitable.¹¹ NetzCope provides a number of visualization tools to vary and compare them. For a quantitative comparison, NetzCope will compute the mutual information¹² of different decompositions.

3. What NetzCope does

For moderately large networks of some 10^4 vertices, say, there is not only the challenge of finding a display which exhibits as much as possible of the network structure. There are simply too many vertices and edges to fit distinguishably into a plot of any reasonable size.

As a consequence, a central part of our strategy will be to identify, display, and analyze communities within the overall network. These (interconnected) communities do admit a graphical representation, and so NetzCope first displays a network of communities. For more detailed analysis the software then allows the user to “zoom into” communities and explore their inner structure.

In contrast to well established network analysis software tools such as UCINET or Pajek, NetzCope implements new methods to analyze and visualize network structures, with a special emphasis on using recent methods from statistical physics to identify and visualize community groups and to analyze and visualize the adjacency matrix.

Overall, the principal functionalities of NetzCope are:

- (1) Identification of disjoint components within the network, if any such exist. For any such component it will perform the following tasks:
- (2) Display of the adjacency matrix which encodes the connectivity
- (3) Display the same for the matrix after “Fiedler ordering” which reorders the original (e.g., alphabetical) network listing in such a way that interacting partners are grouped together

- (4) Generate plots of the nodes and the links between them in two or three dimensions, and rotate the plots about their axes
- (5) Identify *communities* of close collaboration inside the network
- (6) Plot the network of these communities. This achieves one of the main goals namely a suitable complexity reduction, so as to arrive at a feasible and meaningful graphical display of networks of a size where plotting of the full network becomes meaningless. The following items are tools for the further analysis of these communities, such as
- (7) Topical profile: nodes may carry one or more labels. Their frequency within a community is represented by colored segments in the aforementioned plot. A more precise representation of this as a histogram can be called up for each community, comparing their occurrence within the community to overall occurrence in the network.
- (8) A scatter plot displays the number of internal versus external links for the leading players in the community, thus identifying central players and gatekeepers. Centrality is further analyzed by scatter plots which compare *high linkage* (to many partners) and *important linkage* (to important partners) for leading players.
- (9) An important functionality of NetzCope is the possibility to repeat the above tasks for each of the communities. The user may thus analyze the network iteratively, “zooming into” the community structure until a desired level of structural detail is obtained.
- (10) Finally “network portraits” as proposed by Bagrow et al.¹³ are included, mainly to facilitate the comparison of empirical network structure with that of simulations stemming e.g. from random graph or multi-agent models.

Originally designed to analyze and display bipartite networks of organizations collaborating through projects, we have lifted this restriction to make NetzCope more versatile. NetzCope can now also analyze general, unipartite networks.

NetzCope distributions, together with sample network data files, are available for Linux as well as Windows users. NetzCope loads network data in the widely used Pajek format or from an edge list in plain text format. Netzcope can be downloaded at <http://www.physik.uni-bielefeld.de/~strogan/>.

4. Some NetzCope Screenshots

4.1. *The Connected Components*

After a network has been loaded, NetzCope first extracts and lists the connected components. For a bipartite network, each line in Fig. 1 shows the number of edges (“relations”), the number of nodes of type O (“organizations”), the number of nodes of type P (“projects”). Clicking on one of the boxes displayed will open a menu of options for the further analysis of the chosen component.

4.2. *The Adjacency Matrix*

For large networks, NetzCope displays the non-zero entries (edges) of the adjacency matrix as dots. Figure 2(a) shows part of such a matrix for some 5600 vertices. They will in general be more or less randomly distributed, as long as the list of vertices does not group vertices together which are connected by many links.

Fiedler ordering, reordering the graph vertices so that the components of the Fiedler vector are sorted, does just this. Figure 2(b) shows the same segment of the adjacency matrix after Fiedler ordering with its characteristic concentration of dots near the diagonal, i.e., of short links in the listing.

4.3. *Plotting the Graph*

The menu item “2 D plot” will produce a two dimensional representation of the graph. When networks have more than a thousand vertices, the display will become more and more opaque (Fig. 3(a)), and even the NetzCope zoom function will be of limited use (Fig. 3(b)).

4.4. *Plotting the Network of Communities*

Before NetzCope outputs a graph of communities, various settings can be specified, in particular the number(s) of communities for which NetzCope then computes the decomposition, the respective modularities are displayed as in Fig. 4. For the collaborative research networks one observes a typical leveling off of the modularity Q as the number of communities increases. A click on the chosen value point on the display opens a window to prescribe some graphical settings, and then produces a display of the corresponding network of communities, such as in Fig. 5.

4.5. *The Network Portrait*

The “network portraits” proposed by Bagrow et al.¹³ display (Fig. 6) not only the degree distribution but more generally, row by row, the distribution of all shell sizes where the n -th shell of a node in the network consists of all the nodes at distance n —the first row gives the usual degree distribution, while the highest n is equal to the diameter of the network. These portraits display structural features, including the network diameter ($d = 7$ in our example), which, particularly for large networks, are not accessible to direct visual inspection of the latter. They have proven useful to show how real world networks differ from certain random or multi-agent models.

5. Auxiliary Features

NetzCope provides numerous auxiliary features and settings, some of which have already been mentioned in their proper context. As mentioned above NetzCope allows interactive exploration of the network, permitting pointing into the display and reading out the corresponding information at the side. The display has a zooming and rotating capability, both in two and three dimensions. The latter is useful to show possible congruences; network graphs occasionally become alike after a suitable rotation. Graphical details such as colorings and line widths can be chosen.

The search for communities can be done by maximizing modularity with a greedy algorithm^{6,7} and/or BRIM⁸ (for bipartites), and with varying degrees of randomness and preferred numbers of communities. Their display is controlled by the aforementioned Fiedler vector providing one coordinate of the network nodes, and typically the subsequent eigenvector of the normalized Laplacian to provide the second one. NetzCope allows the user to choose other eigenvectors for this purpose, generating different distributions of the nodes within the display.

Starting from a bipartite network of “organizations” and “projects,” NetzCope can generate, e.g., the projected network of organizations only, which are linked if they share projects. The number of projects may additionally be taken into account to produce a *weighted* graph.

One should also keep in mind that the community search may involve random elements. In that case output will then vary from one run to another. NetzCope offers a quantitative control of these variants by calculating their mutual information.

6. Conclusion

We have presented a tour of the features of NetzCope, a software application for the display and analysis of complex networks. Originally created to support investigating specific collaboration networks, NetzCope has become a general purpose tool for network study. NetzCope allows the user to interactively explore networks, especially with regard to organizing the vertices so that the most important relationships in the network can be observed. Despite its lengthy list of features, NetzCope remains in its infancy, offering much potential for future extension.

Acknowledgments

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References

1. A.-L. Barabási and Z. N. Oltvai, *Nature Reviews Genetics* **5**, 101(February 2004).
2. M. E. J. Newman, *Proceedings of the National Academy of Sciences* **103**, 8577 (2006).
3. M. E. J. Newman and M. Girvan, *Physical Review E (Statistical, Nonlinear, and Soft Matter Physics)* **69**, p. 026113 (2004).
4. F. Chung and L. Lu, *Annals of Combinatorics* **6**, 125 (2002).
5. S. Fortunato, *Physics Reports* **486**, 75(February 2010).
6. M. E. J. Newman, *Physical Review E (Statistical, Nonlinear, and Soft Matter Physics)* **69**, p. 066133 (2004).
7. A. Clauset, M. E. J. Newman and C. Moore, *Physical Review E (Statistical, Nonlinear, and Soft Matter Physics)* **70**, p. 066111 (2004).
8. M. J. Barber, *Physical Review E (Statistical, Nonlinear, and Soft Matter Physics)* **76**, p. 066102 (2007).
9. S. Fortunato and M. Barthelemy, *PNAS* **104**, 36 (2007).
10. U. Brandes, D. Delling, M. Gaertler, R. Gorke, M. Hoefer, Z. Nikoloski and D. Wagner, *Knowledge and Data Engineering, IEEE Transactions on* **20**, 172(December 2007).
11. B. H. Good, Y. A. de Montjoye and A. Clauset, *Physical Review E* **81**, 046106+(Apr 2010).
12. L. Danon, A. Díaz-Guilera, J. Duch and A. Arenas, *J. Stat. Mech.* , p. P09008 (2005).
13. J. P. Bagrow, E. M. Bollt, J. D. Skufca and D. ben Avraham, *EPL (Europhysics Letters)* **81**, 68004+(March 2008).

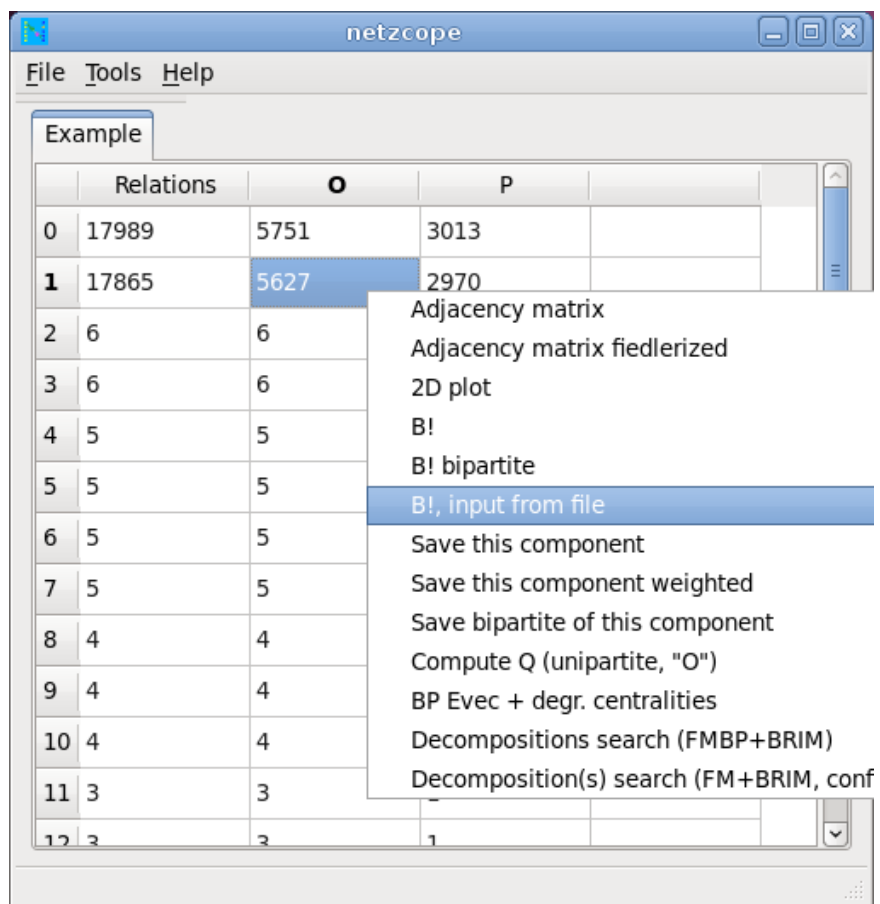
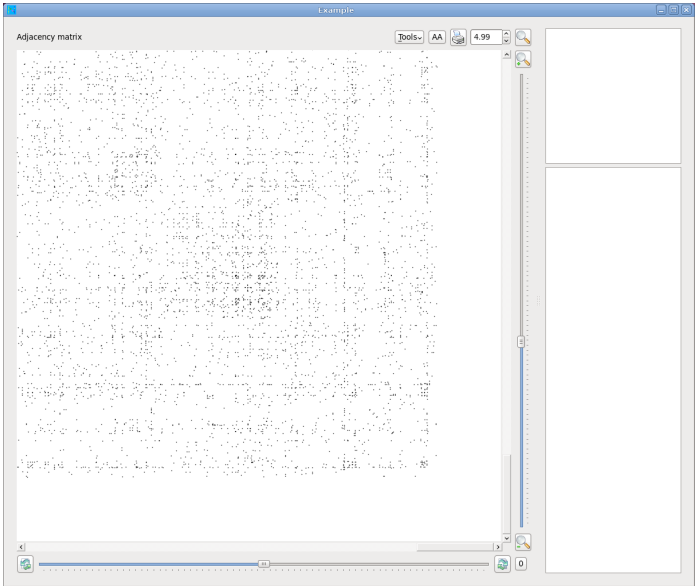
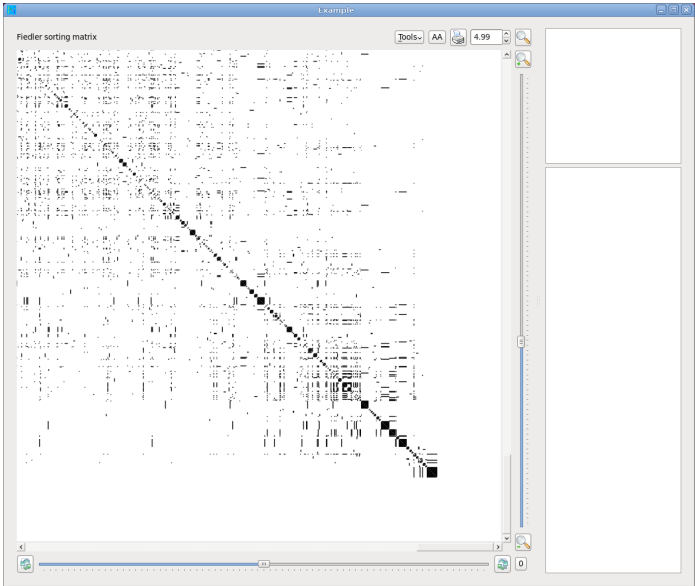


Fig. 1. Connected Components

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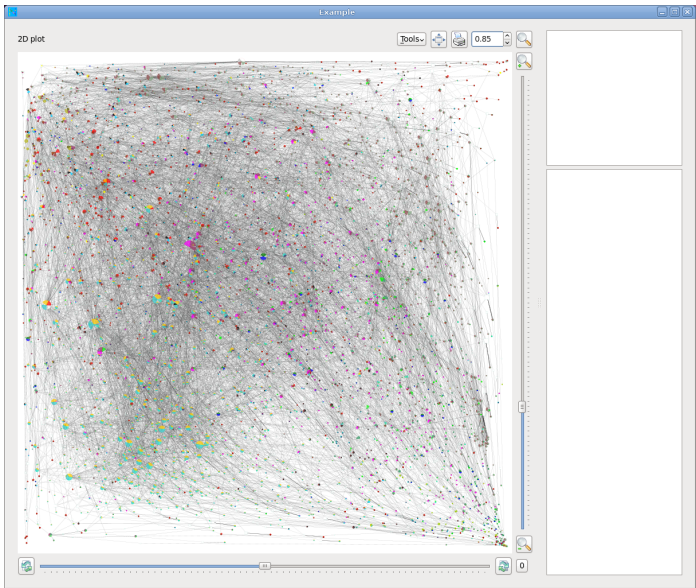


(a) Ad Hoc Ordering

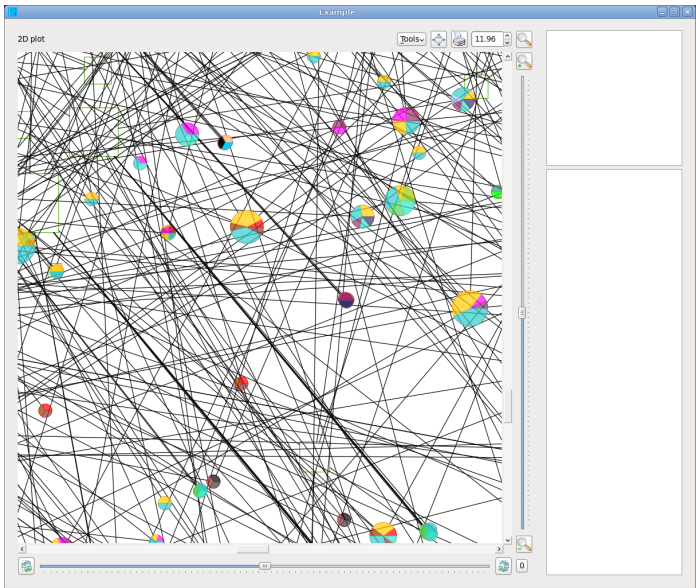


(b) Fielder Ordering

Fig. 2. Adjacency Matrix



(a) The 2-d Plot



(b) Zooming Into the Plot

Fig. 3. Plotting the Graph

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Fig. 4. Modularities vs. Number of Communities

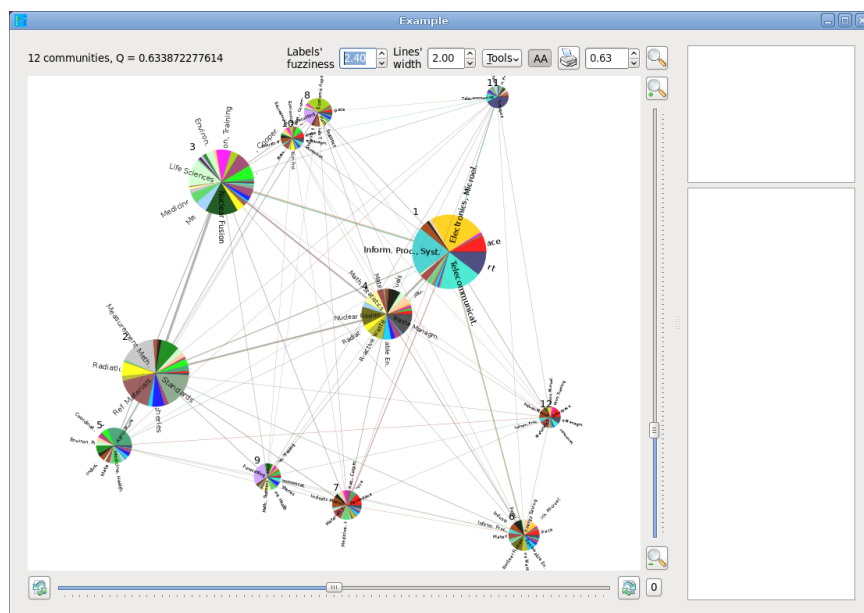


Fig. 5. Network of Communities

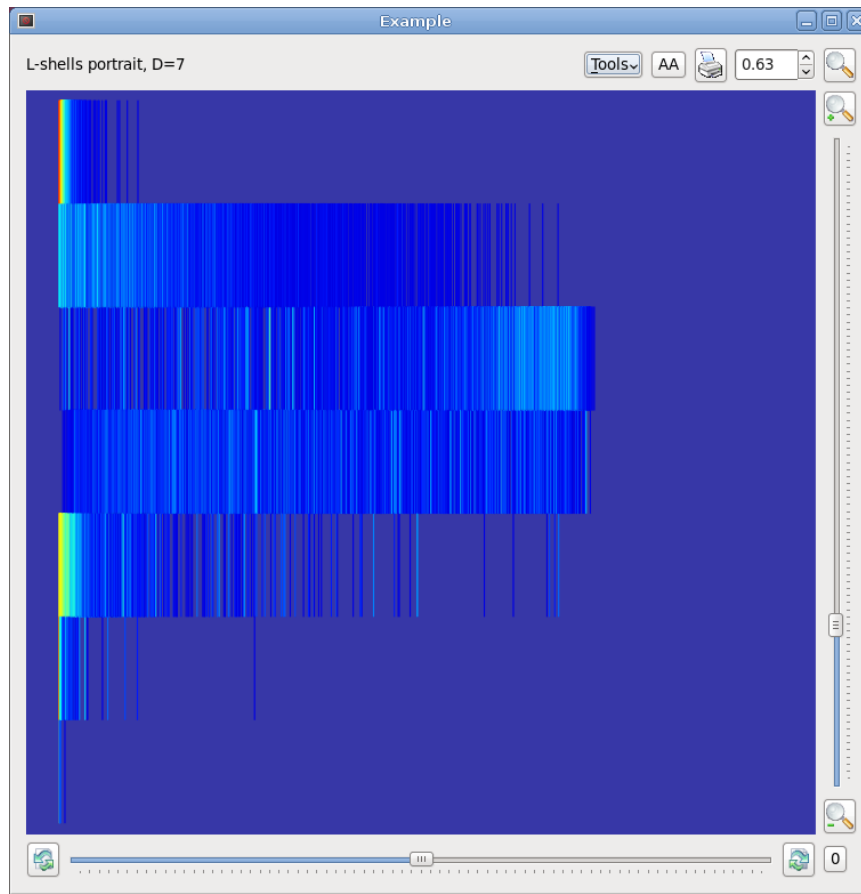
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Fig. 6. Network Portrait